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14. ABSTRACT

Filtering is the recursive estimation of signals observed in noise, a topic of importance in signal processing and other fields which involve the extraction of information from noisy data. The proposal was to investigate filtering using nonlinear, in particular sublinear, expectations. When there is uncertainty about the correct probability describing the noise, a supremum over a class of possible probabilities can be considered, giving a sublinear expectation. In continuous time Peng introduced a G-expectation which is related to a modified Brownian motion

15. SUBJECT TERMS

filtering, signal estimation, sublinear expectation, nonlinear expectation, unknown noise, unknown variance, stochastic control, Nash equilibrium

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UU	UU	UU	UU		Robert Elliott
					19b. TELEPHONE NUMBER: +61-882-9472

Report Title

Filtering using non linear expectations

ABSTRACT

Filtering is the recursive estimation of signals observed in noise, a topic of importance in signal processing and other fields which involve the extraction of information from noisy data. The proposal was to investigate filtering using nonlinear, in particular sublinear, expectations. When there is uncertainty about the correct probability describing the noise, a supremum over a class of possible probabilities can be considered, giving a sublinear expectation. In continuous time Peng introduced a G-expectation which is related to a modified Brownian motion given by the solution of a nonlinear heat equation. Whilst being an interesting concept this definition involves difficult technicalities. An alternative definition of G-Brownian motion uses ideas from stochastic control and considers a supremum over a set of diffusion coefficients.

The first paper completed gives a solution to estimating a Markov chain observed in Gaussian noise when the variance of the noise is unkown. This paper is accepted for the IEEE Transactions on Automatic Control, an A* journal.

The second paper considers the related problem in continuous time. The methods used include stochastic control when the control parameter influences the diffusion coefficients and Nash equilibria from game theory, because the different components of the diffusion can be considered as being controlled by different players. This paper is under a second review for the SIAM Journal on Control and Optimization, an A* journal.

A short third paper discusses how to estimate a change in the transition dynamics of a noisily observed Markov chain. The change point time is hidden in a hidden Markov chain, so a second level of discovery is involved. This paper is accepted for Communications in Stochastic Analysis.

I was quite pleased with the outcomes.

Enter List of papers submitted or published that acknowledge ARO support from the start of the project to the date of this printing. List the papers, including journal references, in the following categories:

(a) Papers published in peer-reviewed journals (N/A for none)

Received Paper

TOTAL:

Number of Papers published in peer-reviewed journals:

(b) Papers published in non-peer-reviewed journals (N/A for none)

Received Paper

TOTAL:

Number of Papers published in non peer-reviewed journals:

(c) Presentations

0

Number of Presentations: 0.00

Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper

TOTAL:

Number of Non Peer-Reviewed Conference Proceeding publications (other than abstracts):

Peer-Reviewed Conference Proceeding publications (other than abstracts):

Received	Paper

TOTAL:

Number of Peer-Reviewed Conference Proceeding publications (other than abstracts):

(d) Manuscripts

Received		Paper
04/16/2016	2.00	Robert Elliott. A NASH EQUILIBRIUM FILTER, SIAM J of Control and Optimization (03 2016)
04/16/2016	1.00	Robert Elliott. Filtering with uncertain noise, IEEE TRANSACTIONS ON Automatic Control (01 2016)
04/16/2016	3.00	Robert Elliott, Sebastian Elliott. HIDDEN MARKOV CHANGE POINT ESTIMATION, COMMUNcations on Stochastic Analysis (11 2015)

TOTAL: **3**

Number of Manuscripts:

Books

Received Book

TOTAL:

Received Book Chapter

TOTAL:

Patents Submitted

Patents Awarded

Awards

Invited to nominate for the Nobel prize in Economics 2015 and 2016.

Published "Stochastic Calculus and Applications" with former student S Cohen. See http://www.springer.com/us/book/9781493928668

Graduate Students

NAME	PERCENT_SUPPORTED

FTE Equivalent:
Total Number:

Names of Post Doctorates

NAME	PERCENT_SUPPORTED

FTE Equivalent:
Total Number:

Names of Faculty Supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Names of Under Graduate students supported

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Student Metrics

This section only applies to graduating undergraduates supported by this agreement in this reporting period

The number of undergraduates funded by this agreement who graduated during this period: 0.00

The number of undergraduates funded by this agreement who graduated during this period with a degree in science, mathematics, engineering, or technology fields:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and will continue to pursue a graduate or Ph.D. degree in science, mathematics, engineering, or technology fields:...... 0.00

Number of graduating undergraduates who achieved a 3.5 GPA to 4.0 (4.0 max scale):...... 0.00

Number of graduating undergraduates funded by a DoD funded Center of Excellence grant for Education, Research and Engineering:...... 0.00

The number of undergraduates funded by your agreement who graduated during this period and intend to work for the Department of Defense 0.00

The number of undergraduates funded by your agreement who graduated during this period and will receive scholarships or fellowships for further studies in science, mathematics, engineering or technology fields:...... 0.00

Names of Personnel receiving masters degrees

NAME
Total Number:

Names of personnel receiving PHDs

NAME
Total Number:

Names of other research staff

NAME	PERCENT_SUPPORTED
FTE Equivalent:	
Total Number:	

Sub Contractors (DD882)

Inventions (DD882)

Scientific Progress

I believe the results are the first for estimating signals observed in noise with uncertain parameters. They consider maxima or suprema of possible probabilities and are reminiscent of some constructions in the Viterbi algorithm

Technology Transfer

None

Report:

Filtering using nonlinear expectations

Foreword

Filtering is the recursive estimation of signals observed in noise, a topic of importance in signal processing and other fields which involve the extraction of information from noisy data. The proposal was to investigate filtering using nonlinear, in particular sublinear, expectations. When there is uncertainty about the correct probability describing the noise, a supremum over a class of possible probabilities can be considered, giving a sublinear expectation. In continuous time Peng [9], [10], introduced a G-expectation which is related to a modified Brownian motion given by the solution of a nonlinear heat equation where the diffusion coefficient involves the supremum over a set of possible coefficients. Whilst an interesting concept this definition involves difficult technicalities. An alternative definition of G-Brownian motion uses ideas from stochastic control and considers a supremum over a set of diffusion coefficients.

The first paper completed gives a solution to estimating a Markov chain observed in Gaussian noise when the variance of the noise is unknown. The methods use maximizing probabilities which are reminiscent of the Viterbi filter. This paper is accepted for the IEEE Transactions on Automatic Control, an A* journal.

The second paper considers a related problem in continuous time. The methods used include stochastic control when the control parameter influences the diffusion coefficients and Nash equilibria from game theory, because the different components of the diffusion can be considered as being controlled by different players. This paper is under a second review for the SIAM Journal on Control and Optimization, an A* journal.

A short third paper discusses how to estimate a change in the transition dynamics of a noisily observed Markov chain. The change point time is hidden in a hidden Markov chain, so a second level of discovery is involved. This paper is accepted for Communications on Stochastic Analysis.

Statement of the problem studied

A basic problem in signal processing is the optimal estimation of a signal observed in noise. A major contribution was provided by Kalman in 1960 who considered linear dynamics with Gaussian noise for the signal and observation processes. His results are credited with playing a major contribution to the successful moon landings of the 196os and 1970s. To estimate Markov chains observed in Gaussian noise the results of Wonham provided answers. The two cases considered by Kalman and Wonham are the only situations (apart from minor extensions) for which closed form, finite dimensional estimates of the noisily observed state are known. Nonlinear filters were widely investigated in the later years of the last century. They were usually implemented by approximations. With improvements in computing power particle filters were popular. These essentially obtain estimates by simulation.

Noise is often modeled by i.i.d. Gaussian random variables, (white noise), in discrete time or by Brownian motion in continuous time. The basic problem discussed in this proposal is how to estimate a Markov chain observed in noise when the statistics of the noise are uncertain. In continuous time the drift of a Brownian motion can be changed using a Girsanov transformation. However, the variance, or quadratic variation, of the Brownian motion noise cannot be changed this way because Brownian motions with different quadratic variations are described by measure which are mutually singular.

To discuss such Brownian motions Peng, [8], [9], introduced solutions of a nonlinear heat equation whose diffusion term is given by a supremum over possible diffusion coefficients. This required much technical mathematics. A more intuitive approach is to consider the supremum over diffusion coefficients as a stochastic control problem. This introduces ideas from stochastic control, but in a nonstandard way because it is the diffusion coefficients which are being controlled.

The proposed problem was to study signals, in particular Markov chains, observed in noise whose parameters are unknown.

Summary of the most important results

The problem of estimating a Markov chain observed in Gaussian noise, where the parameters of the noise are not known, is first solved in discrete time in the paper 'Filtering With Uncertain Noise' which is accepted for publication in the IEEE Transactions on Automatic Control. Sublinear expectations are introduced by considering suprema over possible probabilities at each time step. The methods are extensions of those in our earlier papers and book [2], [3], [5], incorporating ideas of Peng from [8], [9].

A related problem in continuous time is considered in the paper 'A Nash Equilibrium Filter' where we consider the recursive estimation of a finite state Markov chain observed in Gaussian noise. The new feature is that the drift coefficients in the observation process are unknown and time varying.

In the related Zakai equation, which gives the dynamics of the un-normalized conditional probability distribution of the chain, these parameters appear as unknown diffusion terms multiplying a Brownian motion noise.
These unknown terms in the observation process mean that exact conditional expectations cannot be obtained.

When the correct probability measure is uncertain sublinear expectations can again be introduced. These can be defined as suprema over a family of expectations. In the filtering problem this suggests the consideration of the maximization of the components of the Zakai equation with the unknown parameters as control variables. The different components are inter-related, so a related Nash equilibrium is introduced. The control variables appear in the diffusion terms of the Zakai equation and the general maximum principle of Peng, Yong and Zhou [6], [7] and [10], is used. This is framed in terms of first and second order adjoint processes defined by backward stochastic differential equations. It is then extended to describe a Nash equilibrium.

Consequently the content of this work involves several topics: stochastic control theory with the control in the diffusion term, backward stochastic differential equations (BSDEs), non-linear expectations, Brownian motion with uncertain volatility, Nash equilibria and filtering with uncertain parameters. These are related through the discussion of the basic problem, namely the filter which estimates a Markov chain observed in Brownian motion.

The third paper considers the situation where a discrete time Markov chain X is observed indirectly through a second Markov chain Y. However, the dynamics of X undergo a change at a random time. The change point is hidden in the dynamics of the chain X for which only imperfect observations Y are provided. Consequently the change point can be thought of as being hidden at a second level. Nonetheless, given the observed process Y, filtered recursive estimates for the conditional distribution of X and the change point are derived. The techniques are developments of those found in [1], [2] and [5].

Bibliography

[1]. Dembo, A. and Zeitouni, O.: Parameter estimation of partially observed continuous time stochastic processes via the EM algorithm, *Stoch. Proc. Appl.* **23** (1986), no. 1, 91-113.

[2] R.J. Elliott, Exact adaptive filters for Markov chains observed in Gaussian noise*, Automatica* 30 (1994): 1399-1408.

[3] R.J. Elliott and W.P.Malcolm, Data-recursive smoother formulae for partially observed discrete-time Markov chains, *Stoch. Anal. Appl.* **24** (2006), no. 3, 579-597.

[4] R.J. Elliott, L. Aggoun and J.B. Moore, Hidden Markov Models: Estimation and Control, Springer-Verlag, Berlin-Heidelberg-New York, Applications of Mathematics, 29 Corrected Third Printing, 2008, 380 pp.

[5] R.J. Elliott and J. Deng, A Viterbi smoother for discrete state space model, *Systems & Control Letters* 58 (2009), 400-405.

[6] S. Peng and Y. Hu, Maximum principle for semilinear stochastic evolution control systems, *Stochastics and Stochastics Reports*, 33 (1990), 159-180.

[7] S. Peng, Stochastic Hamilton-Jacobi-Bellman equations, *SIAM J. Control* 30(2), (1992) 284-304.

[8] S. Peng, G-Expectation, G-Brownian Motion and Related Stochastic Calculus of Ito's Type, The Abel Symposium 2005, Abel Symposia 2, Editors: Benth et al., Springer-Verlag, 2006, 541-567.

[9] S. Peng, Nonlinear expectations and stochastic calculus under uncertainty, (2010), arXiv:1002.4546.

[10] J. Yong and X.Y. Zhou. Stochastic Controls, Hamiltonian Systems and HJB Equations. Springer. Applications of Mathematics 1999.